RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2017

SECOND YEAR [BATCH 2015-18]

Date : 27/05/2017 **MATHEMATICS FOR ECONOMICS (General)**

Time : 11 am – 2 pm **Paper :** IV Full Marks : 75

[Use a separate Answer Book for <u>each group</u>]

<u>Group – A</u>

Answer **<u>any six</u>** questions from **<u>Question Nos. 1 to 8</u>** : [6X5]

1. Define an Euclidean space. Also define the norm of a vector. Prove that for any two vectors α, β in a Euclidean space *V*, $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$. 3+2

2. a) Find the characteristic polynomial of the following matrix. 3

$$A = \begin{bmatrix} 2 & 1 & 7 \\ 3 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}$$

b) Find the linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ corresponding to the matrix *A* [where matrix A is given in Question No. 2(a)] 2

3. a) Define the geometric and algebraic multiplicity of an eigen value of a matrix *A*. 2+2

b) If the characteristic polynomial of *A* is $f(x) = (x-1)^3(x-2)$ then find det *A*. 1

4. Reduce the following quadratic form to its normal form.

$$x^2 + 5y^2 + 2z^2 - 4xy - 6yz + 2zx.$$

Also find its signature. 4+1

5. Verify Cayley-Hamilton theorem for the matrix. 5

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 4 & 3 \\ 2 & 1 & 1 \end{bmatrix}$$

6. Find the eigen values and eigen vectors of the linear operator. $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by, $T(x_1, x_2, x_3) = (x_1 + x_2, x_3, x_1 + x_3)$. 5

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator such that it rotates every vector in the plane $\mathbb{R}^2$ by an angle θ, where $0 < \theta < \frac{\pi}{2}$. Does *T* have any real eigen vector? Explain. 5

8. Diagonalize, if possible $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 7 & 2 \\ 2 & 3 & 2 \end{bmatrix}$. 5

<u>Group – B</u>

Answer **<u>any one</u>** question from **<u>Question Nos. 9 & 10</u>**: [1X5]

9. a) State the fundamental theorem of Linear Programming. 2

b) Show that the set of vectors (2, 1, 4), (1, –1, 2), (3, 1, –2) form a basis for E^3. 3

10. Transform the following Linear programming problem into the standard maximization form. Show the necessary steps. 5

Maximize $z = 2x_1 + x_2 - 6x_3 - x_4$

Subject to : $3x_1 + x_4 \le 25$

$x_1 + x_2 + x_3 + x_4 = 20$

$4x_1 + 6x_3 \ge 5$

$2 \le 2x_1 + 3x_3 + 2x_4 \le 30$

$x_1, x_2, x_4 \ge 0$ and x_3 is unrestricted in sign.

Answer **any two** questions from **Question Nos. 11 to 14** : [2X10]

11. a) Solve the following L.P.P. graphically.

Maximize $z = 2x_1 + x_2$

Subject to : $x_1 - x_2 \ge 0$

$-2x_1 + 3x_2 \le 6$

and $x_1, x_2 \ge 0$ 4

b) Solve the following linear programming problem by penalty method.

Maximize $z = -2x_1 + x_2 + 3x_3$

Subject to : $x_1 - 2x_2 + 3x_3 = 2$

$3x_1 + 2x_2 + 4x_3 = 1$

and $x_1, x_2, x_3 \ge 0$ 6

12. a) Use two phase simplex method to show that the following linear programming problem has unbounded solution.

Maximize $z = 2x_1 + 3x_2 + x_3$

Subject to : $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

and $x_1, x_2, x_3 \ge 0$ 8

b) Define degenerated and non-degenerated BFS (Basic Feasible Solution) of an L.P.P. 2

13. a) Find the dual problem of the following linear programming problem.

Maximize $z = x_1 + 4x_2 + 3x_3$

Subject to $2x_1 + 3x_2 - 5x_3 \le 2$

$3x_1 - x_2 + 6x_3 \ge 1$

$x_1 + x_2 + x_3 = 4$

$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign. 6

b) Prove that the dual of the dual is the primal. 2

c) What do you mean by linearly independent vectors? 2

14. a) Use duality to solve the following problem.

Maximize $z = 2x_1 + 3x_2$

Subject to $-x_1 + 2x_2 \le 4$

$x_1 + x_2 \le 6$

$x_1 + 3x_2 \le 9$

and $x_1, x_2 \ge 0$ 8

b) What do you conclude on nature of solution: 2

(i) for primal, if primal has feasible solution but dual has no feasible solution?

(ii) for dual, if primal has no feasible solution but dual has feasible solution?

Group – C

Answer **any two** questions from **Question Nos. 15 to 19** : [2X5]

15. Consider the following game where player I has two possible strategies – Up and Down and player II has two possible strategies – Left and Right. 5

		Player II	
		Left	Right
Player I	Up	40, –40	50, –50
	Down	90, –90	10, –10

Find the Nash Equilibrium of the game.

16. Suppose there are two firms in an Oligopolistic market facing the market demand function:

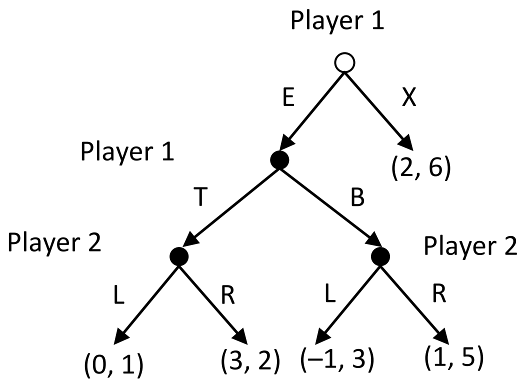
$$\begin{aligned} P(Q) &= \alpha - Q \quad \text{if } Q \le \alpha \\ &= 0 \qquad\quad \text{if } Q > \alpha \end{aligned} \quad \text{where } Q = q_1 + q_2$$

With q_1 the output of firm 1 and q_2 the output of firm 2. Both the firms face the same marginal cost '*e*'. Find out the equilibrium values of output of the firms if:

(i) Both the firms choose their output simultaneously

(ii) Firm 1 has the option to set its output earlier. 2½+2½

17. Find out the subgame Perfect Nash Equilibrium of the following game: 5



Answer **any one** question from **Question Nos. 18 & 19** : [1X10]

18. Show that in a two person zero sum game there is always a Nash Equilibrium in mixed strategies. 10

19. a) Distinguish between a Normal form representation of a game and extensive form representation of a game. 3

b) Compare and contrast between dominant and dominated strategies using appropriate examples. 3

c) Define the concept of Nash Equilibrium in mixed strategies. How is the definition modified in the context of sequential games? 2+2

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